Packet TDEV, MTIE, and MATIE
- for Estimating the Frequency and Phase Stability of a Packet Slave Clock

Antti Pietiläinen

Expressing performance of clocks

TIE = Time interval error

MTIE = Maximum time interval error

MTIE/τ = Maximum frequency error estimate from MTIE

TDEV = Time deviation

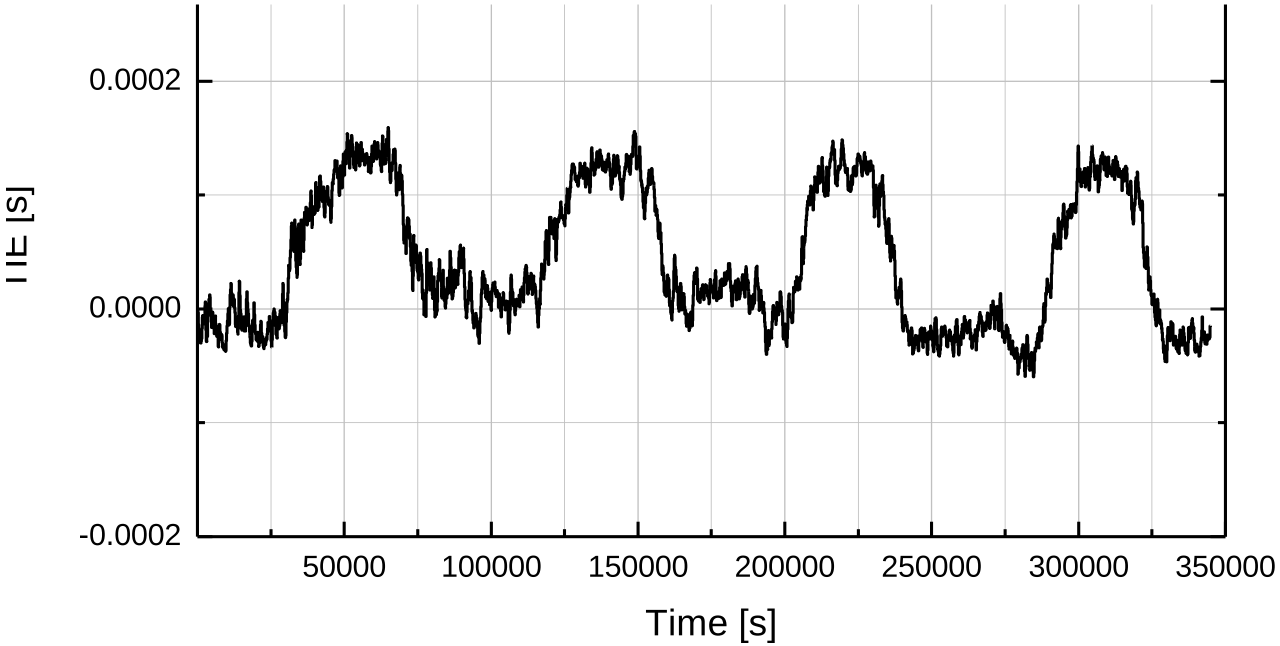
MDEV = Modified Allan deviation

MATIE = Maximum average time interval error (proposal)

MAFE = Maximum average frequency error (proposal)

Time interval error (TIE)

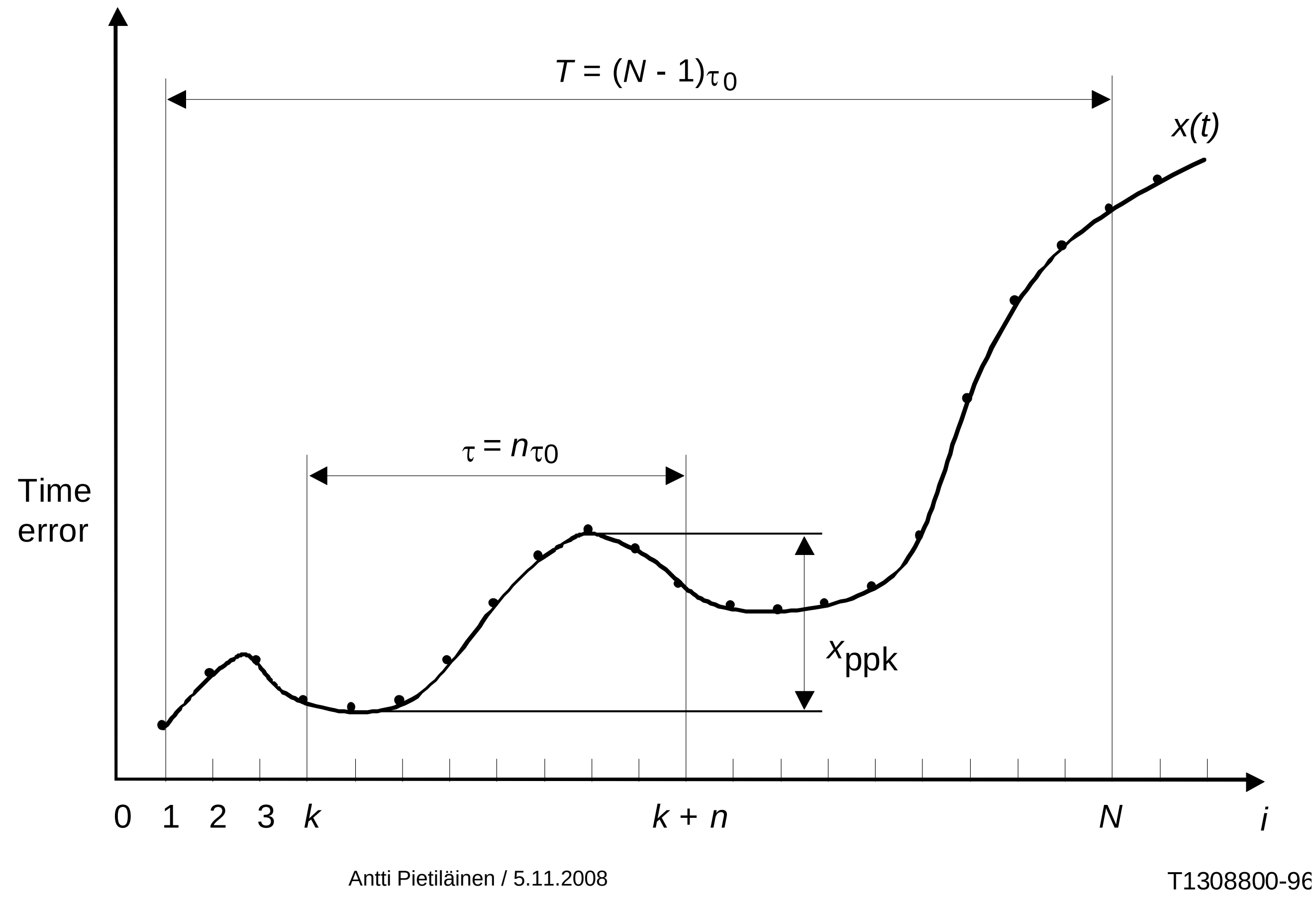
- Indicates the phase of a clock compared with the phase of a reference clock. For example if the frequency of a clock is 10 ppb too large, TIE will increase by 10 ns every second.
- The quantities considered of interest in standardization bodies for characterization of time and frequency stability are calculated from TIE, e.g. MTIE, TDEV, and MDEV.



TIE of a packet clock synchronized over a DSL connection.

MTIE - maximum time interval error

- MTIE defines maximum wander within an observation window. The observation window is slid over the TIE data.
- The sizes of the observation windows are indicated by the x-axis values of the MTIE curve.
- MTIE is specified in G.810. MTIE masks have been specified, for example, in G.812, G.813, G.823, G.824, and G.8261

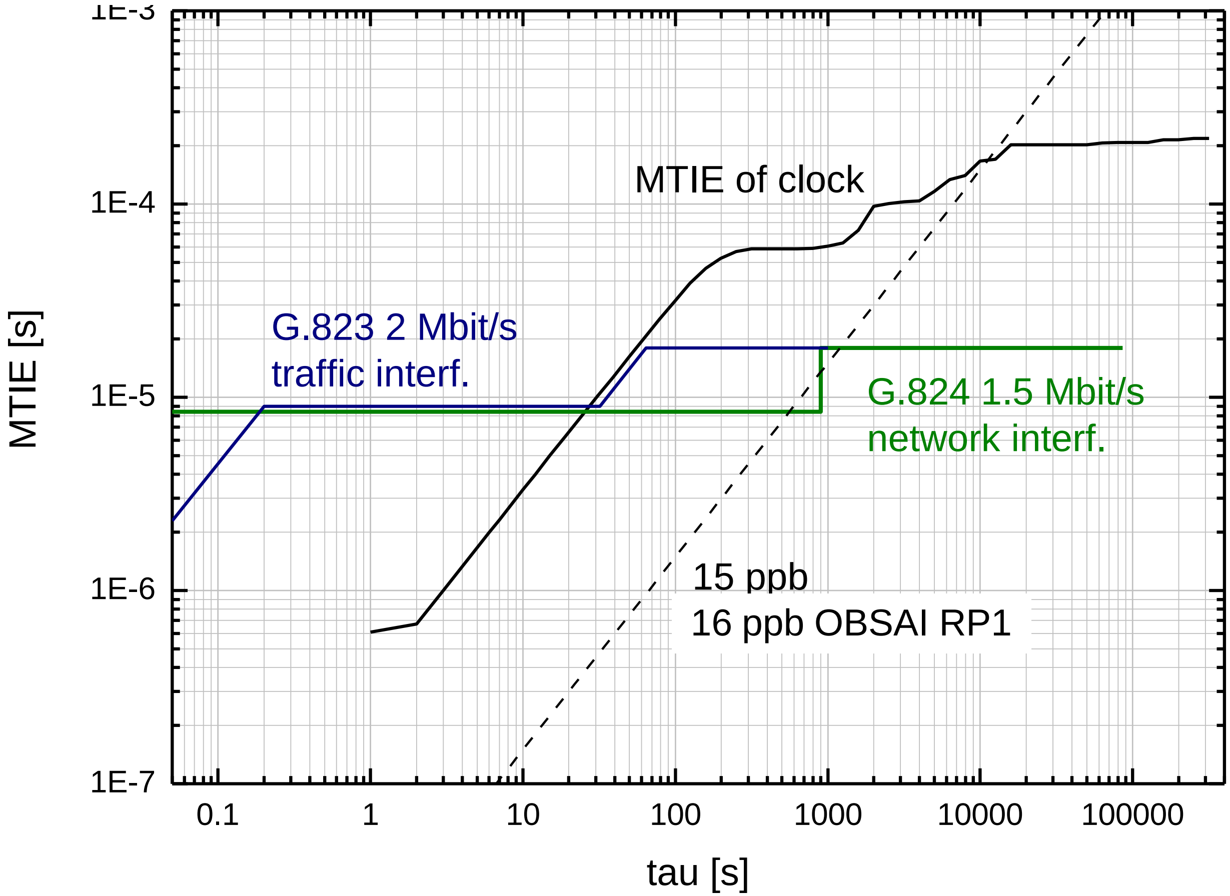


(from G.810)

Nokia Siemens Networks

MTIE of a practical packet clock

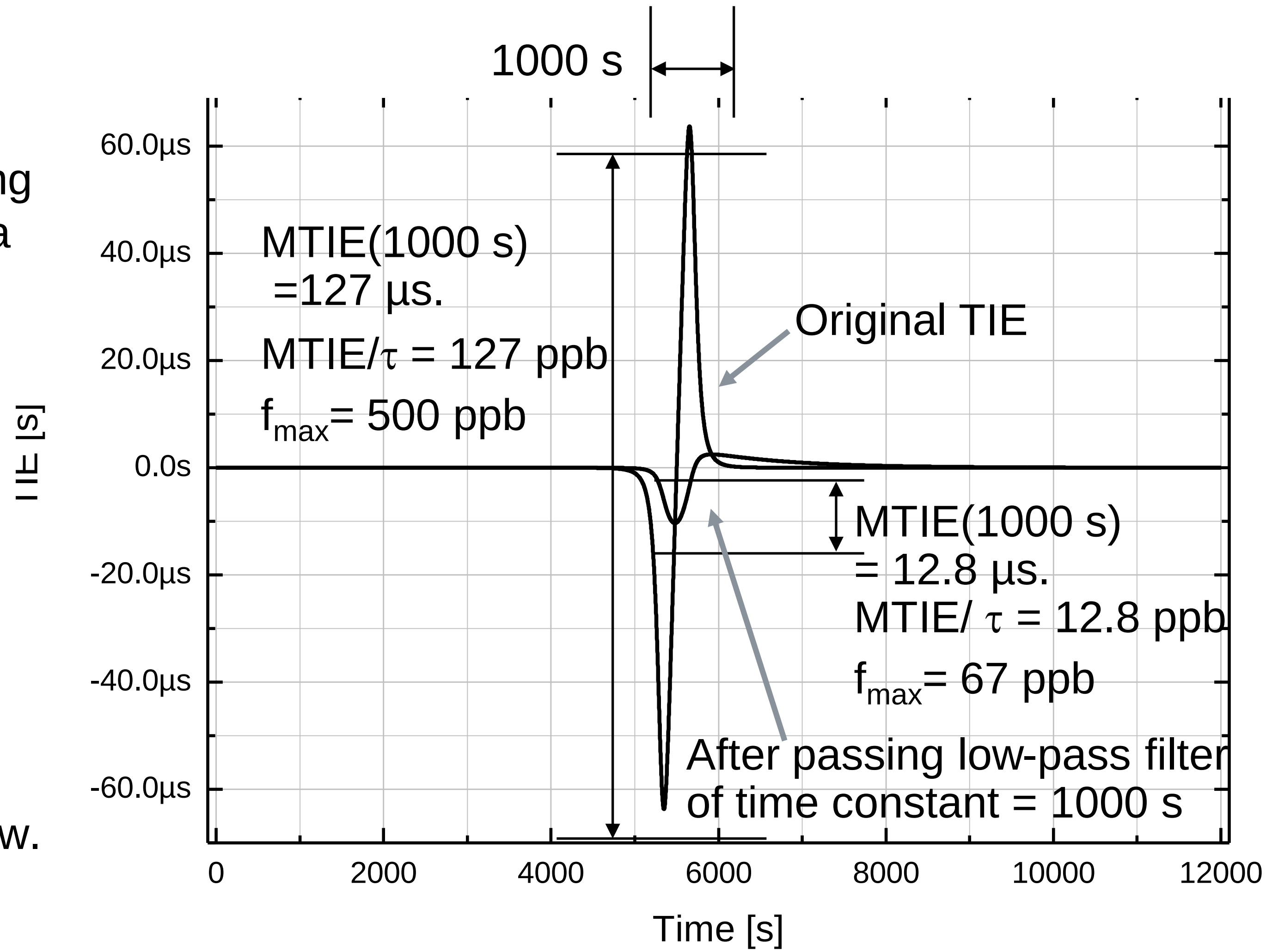
- Each observation window scanned over the TIE curve (see previous slide) will produce one point on the MTIE or MRTIE curve.
- MTIE of a TCXO based packet clock synchronized over a production network and DSL connection shoots about one order of magnitude over the PDH specifications.
- Does one need to average this clock for 10000 seconds to go below 16 ppb?



MTIE of a packet clock synchronized over a DSL connection.

MTIE is a pessimistic estimator when used to indicate achievable frequency stability

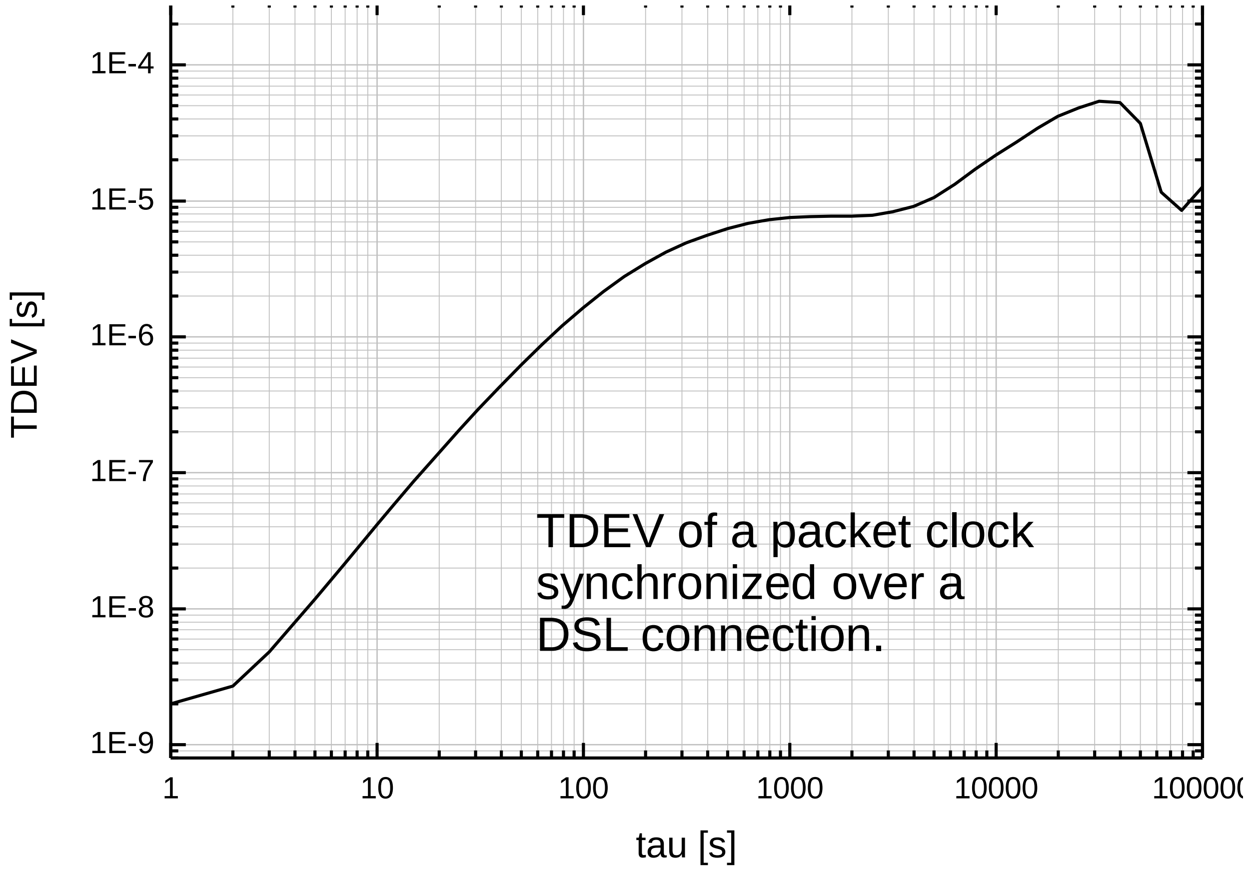
- MTIE predicts a stability of 127 ppb.
- However, by passing the signal through a 1st order low-pass filter with time constant of 1000 s, the maximum frequency error drops to 67 ppb.
- With a better filter, the frequency error could be further reduced without widening the window.



Time deviation (TDEV)

- TDEV indicates phase variation of a clock as a function of averaging time.
- TDEV masks have been specified in various ITU-T recommendations.
- TDEV averages out extremes.
- TDEV and MDEV (on following slide) are closely related, see formula.

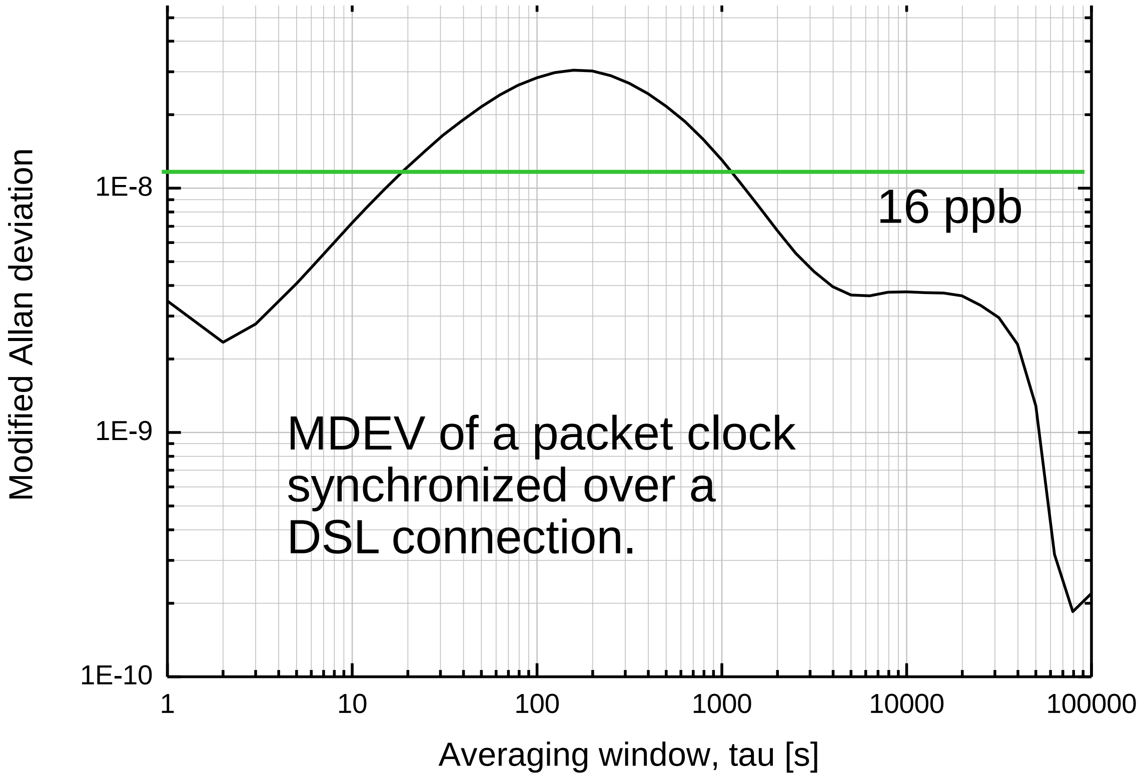
$$\mathrm{MDEV} = \frac{\sqrt{3}\,\mathrm{TDEV}}{\tau}$$



Nokia Siemens Networks

Modified Allan deviation (MDEV)
(root of modified Allan variance)

- Is used to indicate frequency variation and frequency uncertainty of clocks.
- Because frequency variation is built up from various noise phenomena, the fundamental accuracy limits of a clock can be determined by averaging the clock frequency over a period of time. Consequently, Allan variance is indicated in graphs as a function of averaging window size.
- Allan variance is an averaging function and hides occasional bad performance.

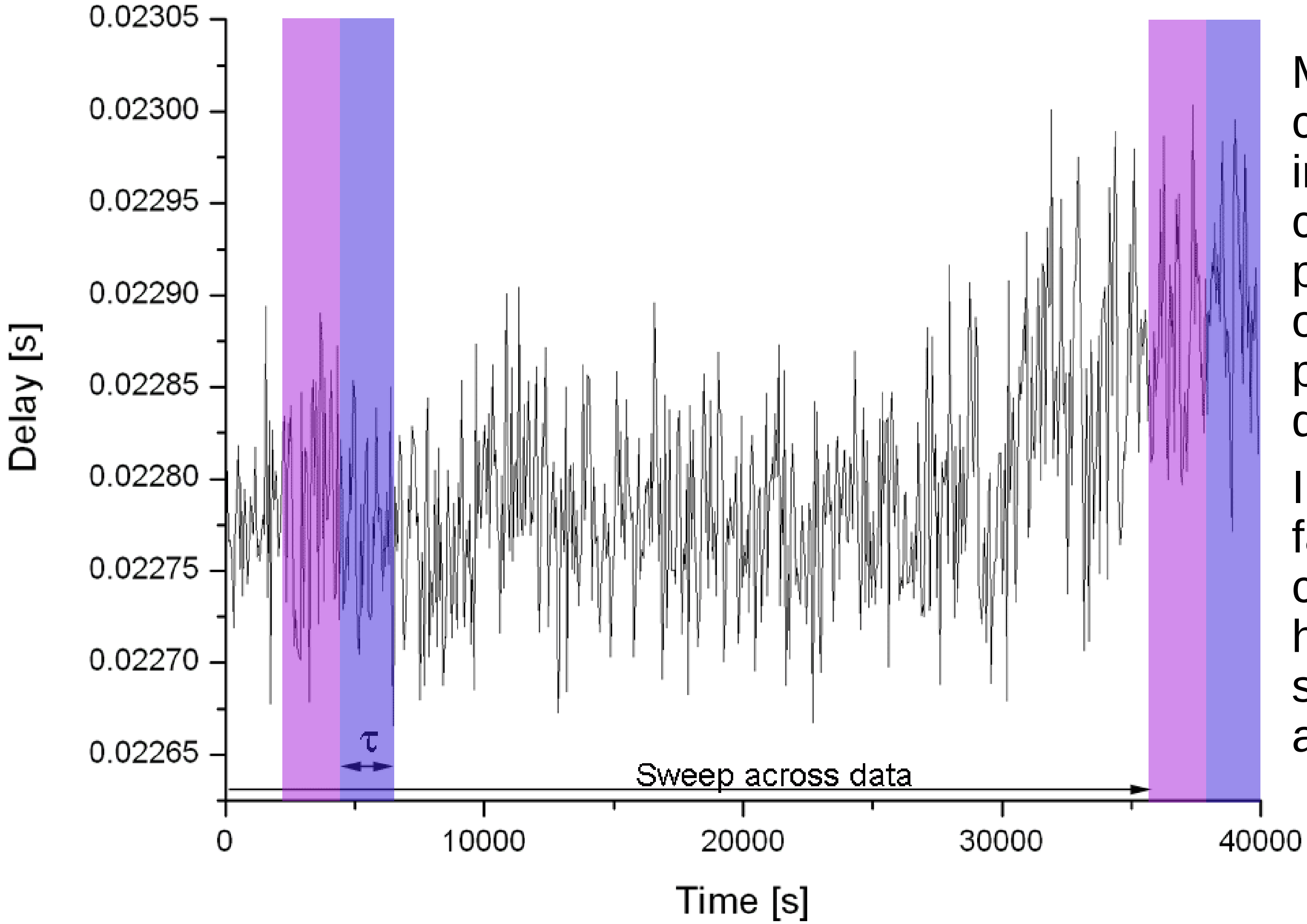


MATIE (maximum average time interval error), MAFE (maximum average frequency error)

- A new metric has to be defined.
- The target: a metric that
  - Describes the upper bound of the phase or frequency error of a clock.
  - Is presented as a function of averaging window width, similar to TDEV or MDEV. The averaging window size at which the metric drops below required level corresponds to the required local oscillator stability.
  - Can be calculated from packet delay of fastest packets and from the phase (TIE) of a clock.
- Solution :
  - Calculate average phase (or packet delay) difference between two windows next to each other.
  - Find the maximum value of the difference over the whole data.

Calculating Maximum average time interval error

- Calculate x_i = abs[average(blue)-average(purple)]
- Slide the windows over the data and find max(x_i)
- Change the window size and start over until all window sizes of interest have been covered.



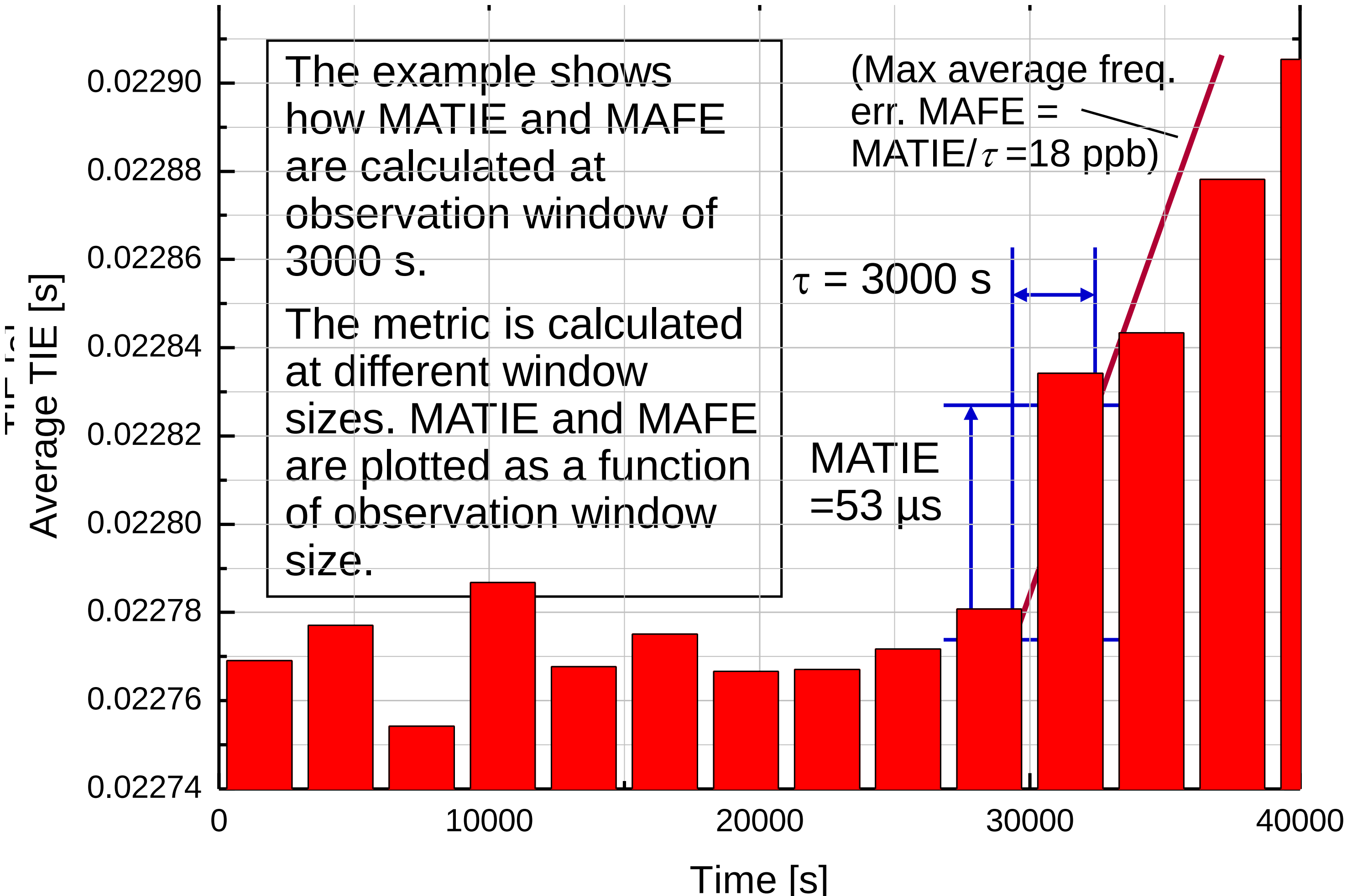
MATIE can be calculated from time interval error (TIE) of a clock, as well as from packet delays. In case of packet delays, a percentile selection is done first.

In the figure, 10 fastest of each 1000 consecutive samples have been pre-selected and averaged.

Determining the maximum average time interval error MATIE and max. average frequency error MAFE at τ = 3000s.

TIE (time interval error) is averaged over observation window and maximum change between two consecutive windows is determined.

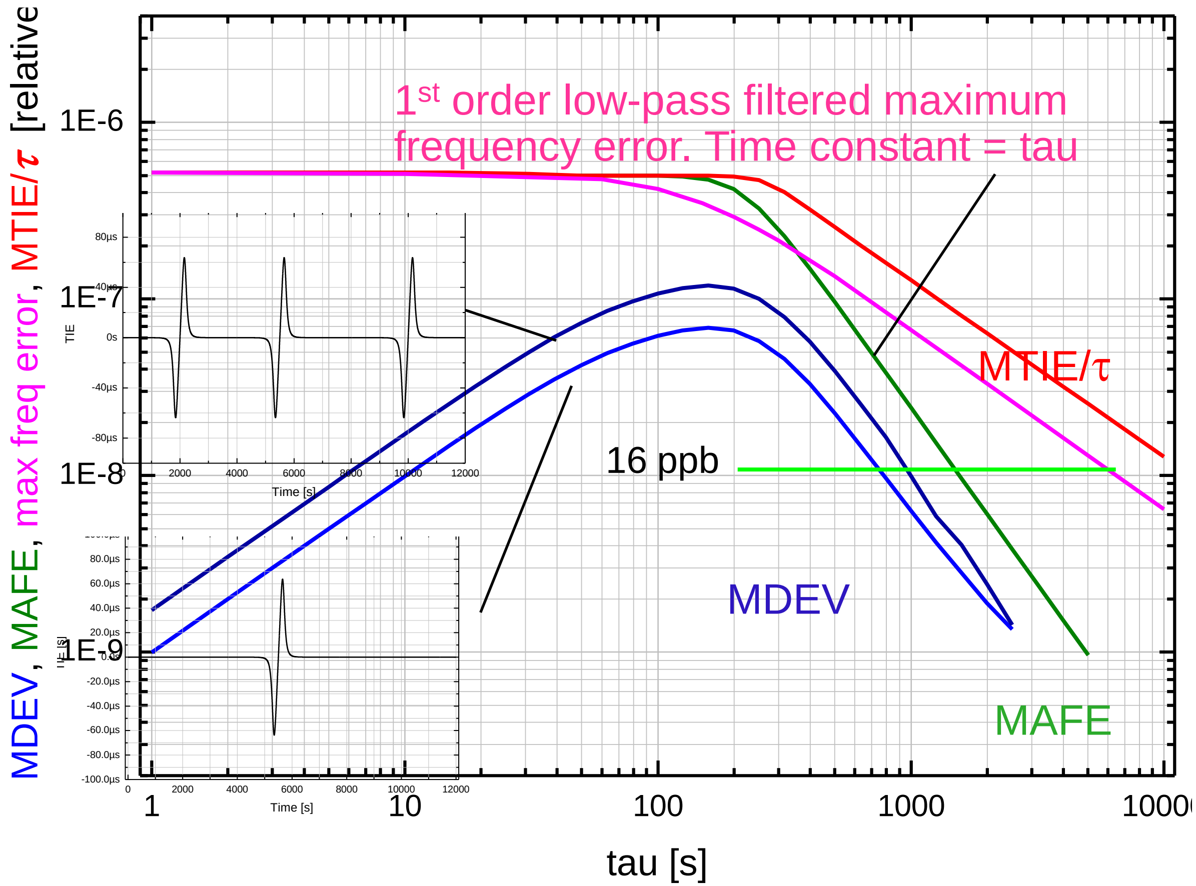
Note, this is a simplification. In reality the adjacent averaging windows slide over the data.



The function estimates achievable worst-case stability over the whole TIE measurement as a function of the averaging time of the clock algorithm.

Comparison between frequency stability estimations of MDEV, MTIE/τ, and MAFE
(corresponds to TDEV, MTIE, and MATIE in phase domain)

- MDEV varies depending on fill ratio and is optimistic.
- MTIE/τ is too pessimistic.
- The different metrics estimate very different frequency stability.
- It is expected that low-pass filtered max. frequency error will approach MAFE when the filter is more optimal.



Nokia Siemens Networks

MATIE follows quite closely the framework set by TDEV and MTIE

$$\mathrm{MATIE}(n\tau_0) \cong \max_{1\le k\le N-2n+1} \frac{1}{n} \left| \sum_{i=k}^{n+k-1} (x_{i+n} - x_i) \right| \qquad , n = 1, 2, ..., \text{integer part } (N/2)$$

$$\mathrm{TDEV}(n\tau_0) \cong \sqrt{\frac{1}{6n^2(N-3n+1)} \sum_{j=1}^{N-3n+1} \left[\sum_{i=j}^{n+j-1} (x_{i+2n} - 2x_{i+n} + x_i) \right]^2} \quad , n = 1, 2, ..., \text{integer part } \left(\frac{N}{3}\right)$$

$$\mathrm{MTIE}(n\tau_0) \cong \max_{1\le k\le N-n} \left[\max_{k\le i\le k+n} x_i - \min_{k\le i\le k+n} x_i \right] , \qquad n = 1,2,\ldots, N-1$$

$n\tau_0$ is the observation window length, n is the number of samples in the window, τ_0 is the sample interval, N is the number of samples in the data set. Index variable i is incremented to scan across the window and k or j is incremented for sliding the window.

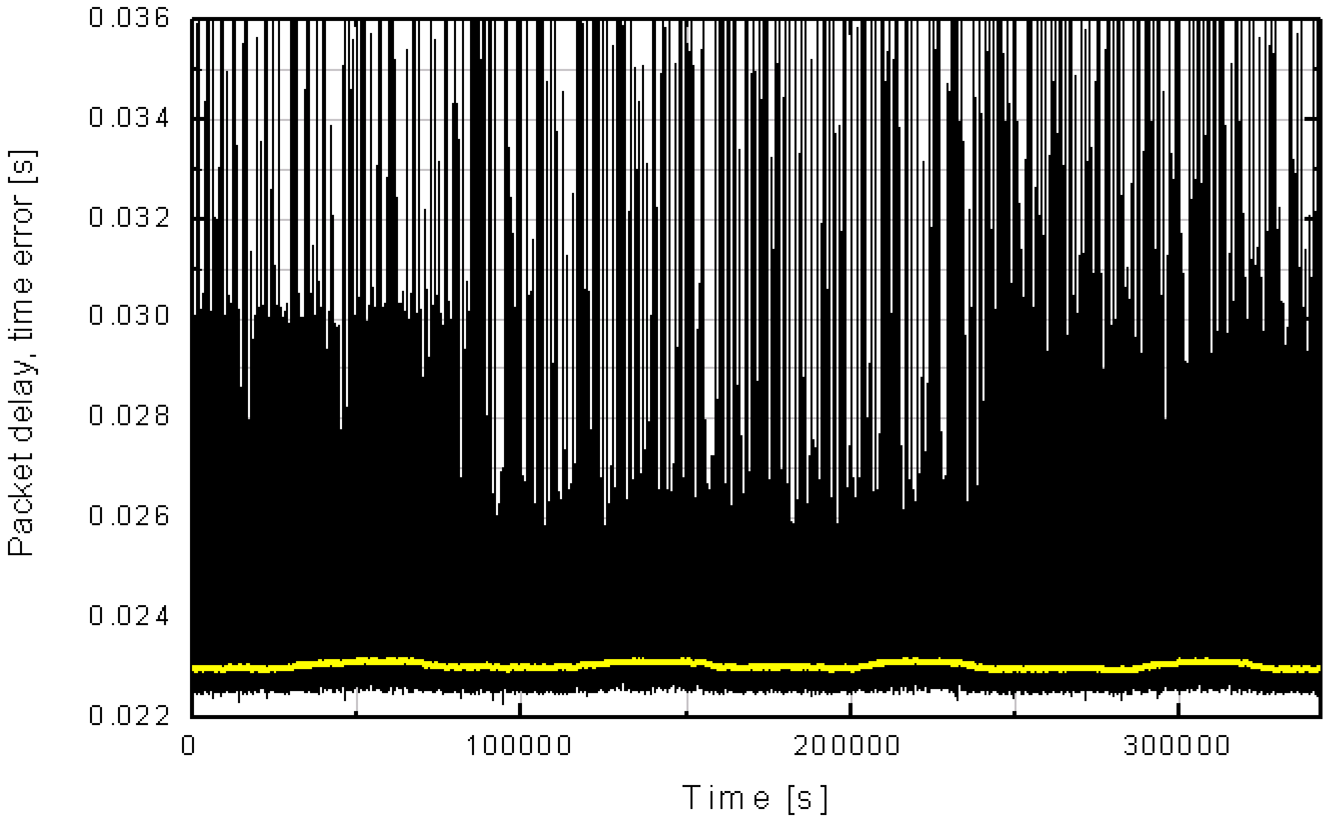
Nokia Siemens Networks

Estimating packet clock performance from packet delay variation

Packet delay variation is not a good indicator of expected packet clock properties because the time interval error of a packet clock is a small and unknown fraction of the packet delay variation.

Black: Timing packet delay

Yellow: TIE of the packet clock

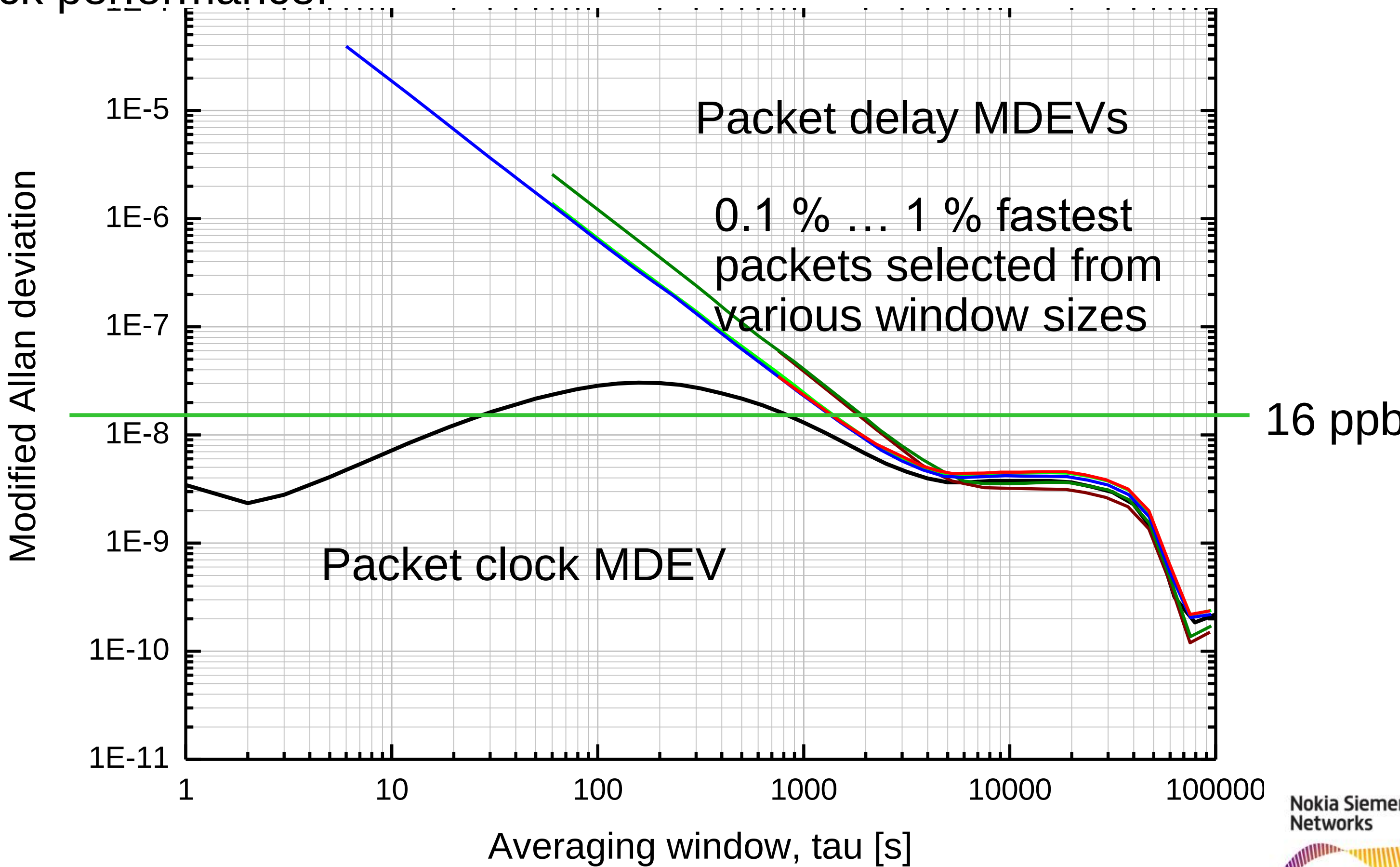


Describing packet networks' capability to support packet timing: Proposals

Measurement	Author	'Published'	Time
Minimum TDEV	Symmetricom	ITU contrib. accepted	June 2007
Minimum picking TDEV	Semtech	ITU contrib.	June 2007
Percentile TDEV	Symmetricom	No public or standards documents	Taken into discussion in ~December 2007
Fixed selection window percentile TDEV & MDEV	Nokia Siemens Networks	Not published	
Fixed selection window percentile MTIE	Nokia Siemens Networks	ITU contrib.	May 2008
MATIE, MAFE	Nokia Siemens Networks	ITU contrib.	Sep 2008

Comparing modified Allan deviation of packet clock and packet MDEV of delay values

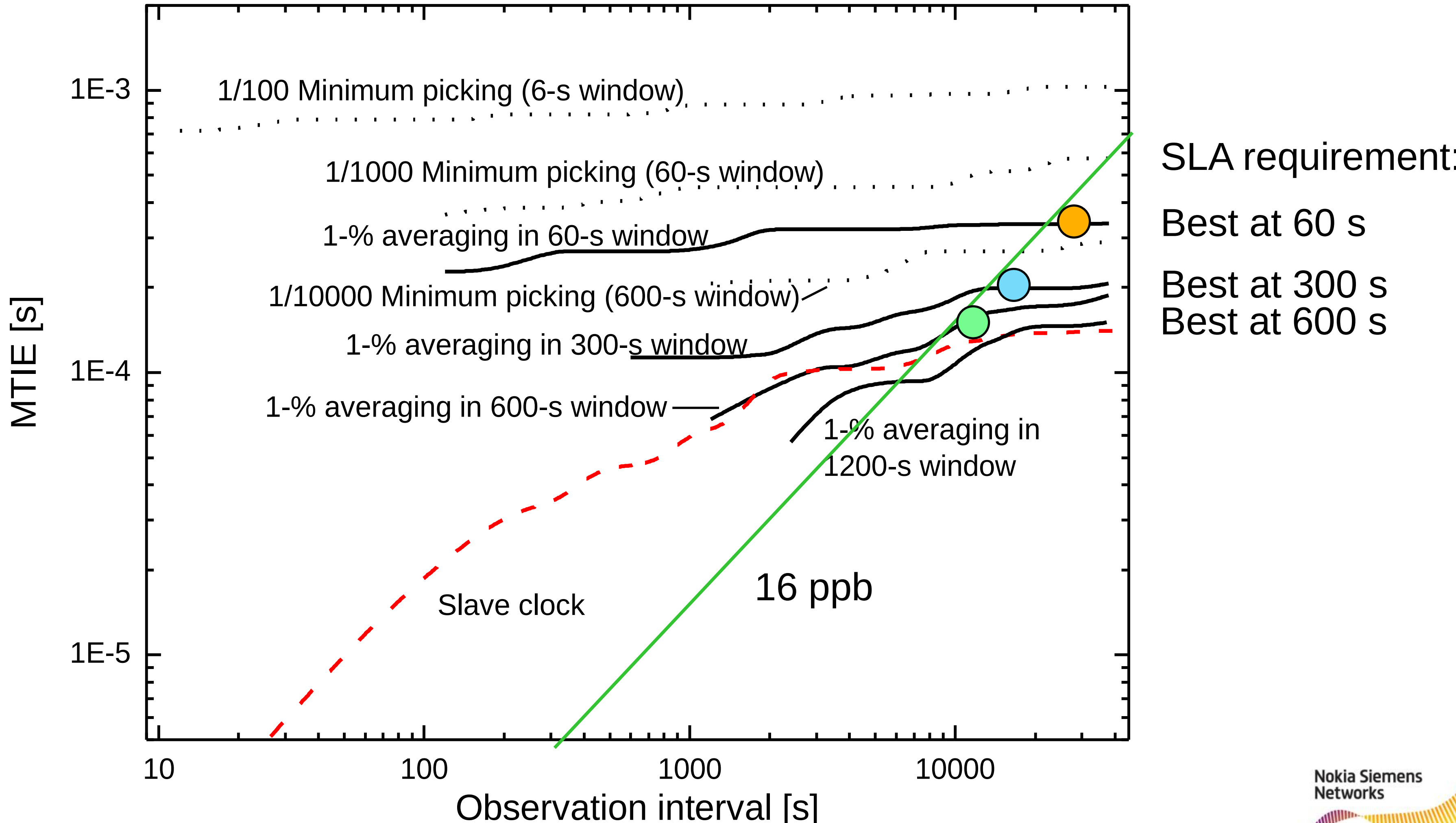
- At small averaging windows the short-term stability of the local oscillator dominates the packet clock performance.
- At certain point the curves almost combine.
- TDEV (and MDEV) first showed the link between packet delay variation and packet clock performance!



Nokia Siemens Networks

Packet MTIE vs. packet clock MTIE

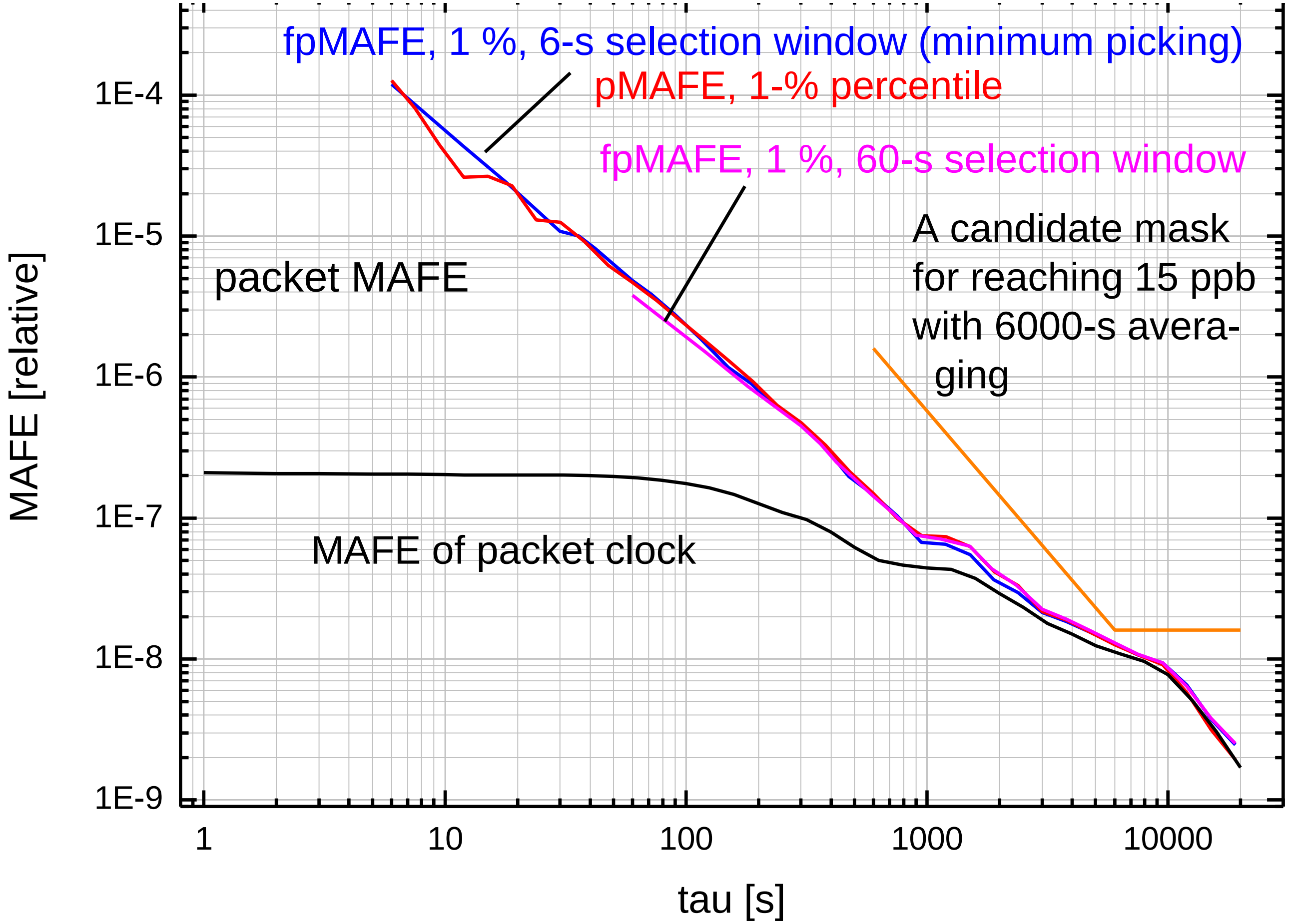
- Packet MTIE would allow setting a performance limit mask.
- However, in addition to being pessimistic considering frequency stability, it is difficult to find parameters for packet MTIE that would accurately describe the performance of a packet clock.



Nokia Siemens Networks

Maximum average frequency error (MAFE)

- Regardless of the exact packet selection method used in calculating packet MAFE the curves remain within a reasonably small range.
- The packet MAFE and MAFE of the clock coincide at large averaging windows .



Conclusions of packet timing metrics

- The introduction of packet TDEV in the form of minTDEV was the long-awaited breakthrough in correlating packet delay variation with packet clock performance.
- TDEV and MDEV describe average performance. Therefore, not accurately usable as limit values when occasionally protruding packet delay variations determine the boundary values of the performance.
- MTIE has been used traditionally also for defining frequency accuracy limits. However, in this use gives usually worse estimation than actual performance, incorporating thus unnecessary and variable “safety margin”. Further, it is difficult to match packet MTIE with packet clock MTIE.
- MATIE and MAFE seem to avoid these issues and the first analysis considering them as performance estimators and as limiting values have been promising.
- More analysis with various TIE data and reference slave clocks are still needed.

Backup – formulas as pictures

p. 7. $$\mathrm{MDEV} = \frac{\sqrt{3}\,\mathrm{TDEV}}{\tau}$$

p. 13.

$$\mathrm{MATIE}(n\tau_0) \cong \max_{1\le k\le N-2n+1} \frac{1}{n}\left|\sum_{i=k}^{n+k-1}(x_{i+n}-x_i)\right| \quad , n = 1, 2, \ldots, \text{integer part } (N/2)$$

$$\mathrm{TDEV}(n\tau_0) \cong \sqrt{\frac{1}{6n^2(N-3n+1)}\sum_{j=1}^{N-3n+1}\left[\sum_{i=j}^{n+j-1}(x_{i+2n}-2x_{i+n}+x_i)\right]^2}\ , n = 1, 2, \ldots, \text{integer part } \left(\frac{N}{3}\right)$$

$$\mathrm{MTIE}(n\tau_0) \cong \max_{1\le k\le N-n}\left[\max_{k\le i\le k+n} x_i - \min_{k\le i\le k+n} x_i\right], \quad n = 1,2,\ldots,N-1$$

$n\tau_0$ is the observation window length, n is the number of samples in the window, τ_0 is the sample interval, N is the number of samples in the data set. Index variable i is incremented to scan across the window and k or j is incremented for sliding the window.