ACS Timing Product Group
Network Metrics

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Presentation Outline

• Rationale for Network Metrics: Can the network support synchronization? If so, what quality level can be supported?
  - Underlying principles of timing over packet networks
    • Theoretical basis
    • Primary cause of inaccuracy is packet delay variation (PDV)

• Metrics
  - TDEV, min_TDEV
  - Spread-separation

• Examples
  - Gamma density function (white noise)
  - 8-switch simulation with different loads (based on G.8261)

• Approximations of min_TDEV
Timing over packet - principle

Slave time offset: \( t = \tau + \epsilon \)

\( RTD = \) round-trip delay

\( \epsilon = \left( \frac{1}{2} \right) (t_4 - \tau_3 - \tau_2 + t_1) \)

\( RTD = (t_4 - \tau_3 + \tau_2 - t_1) \)

Assumption: \( \Delta_{MS} = \Delta_{SM} = \left( \frac{1}{2} \right) \cdot (t_4 - \tau_3 + \tau_2 - t_1) = \left( \frac{1}{2} \right) \cdot RTD \)

Error: \( \Delta \epsilon = \left( \frac{1}{2} \right) (\Delta_{MS} - \Delta_{SM}) \)

Time offset estimate error caused by asymmetry in transit delay

Frequency offset estimate based on two packets (one-way):

\[
\Delta \hat{y} = \frac{(t_1(n_2) - t_1(n_1)) - (\tau_2(n_2) - \tau_2(n_1))}{(t_1(n_2) - t_1(n_1))} = \Delta y - \frac{(\Delta_{MS}(n_2) - \Delta_{MS}(n_1))}{(t_1(n_2) - t_1(n_1))}
\]

Frequency offset estimate error caused by packet delay variation
Timing transfer without intermediate clocks

Master

ToD + 1PPS

Slave

RTT

PDV(sync)

PDV(DReq)

Graphs showing PDV distribution.
PDV is not always so easy to work with......
Network Measurement

“Master side”

CLK

source

Time Stamp

Δ_MS

Packet Network

“Slave side”

CLK

destination

REF

Source packets: TOD nominally uniformly spaced

At destination: packet TOA = TOD + min_delay + PDV
Network metrics

- Based on measurements of transit delay
  - $\Delta_{MS}(n) = \text{transit delay of } n^{th} \text{ packet}$
  - $\Delta_{MS}(n) = \Delta_0 + x(n)$
    - $\Delta_0 = \text{minimum delay (propagation)}$
    - $x(n) = \text{packet delay variation (aka IPDV) (non-negative random variable)}$
  - Assume that inter-packet interval is (approximately) constant
    - $\{x(n)\} = \{x(nT_0)\}$ (discrete-time signal with implied sampling rate of $f_0 = 1/T_0$)

- Errors caused by non-zero nature of $\{x(n)\}$:
  - Error in estimation of minimum delay ($\Delta_0$) affects time
  - Variability of $x(n)$ affects frequency (and time)
    - White noise component can be reduced by averaging
    - Oscillator performance restricts averaging time
    - Short-term slope in $\{x(n)\}$ affects slave frequency offset estimate

- Metrics quantify the “non-zero” nature of $\{x(n)\}$
  - Permissible to ignore (some) packets deemed to have excessive delay variation
Network Metrics (spread-separation)

- Intent of spread-separation metrics:
  - Identify the extent to which packets that have excess delay variation can be discarded
  - Retained packets are not uniformly spaced in time

- Basis: For a given $\Delta$, discarding packets with delay $> \Delta$ is permissible provided that
  - The number of packets retained is statistically significant.
  - The time interval over which there are no packets retained is sufficiently small. That is, there should not be large time gaps in the non-discarded data.

- Given a data set of $N$ points, $\{x_i; i = 0,1,2,...,(N-1)\}$ with implied sampling interval $T_0$ with $\min[x_i]=\Delta_0$:
  - Construct the set $\{i_k; k=0,1,...,(K-1)\}$ defined by $\Delta_0 \leq x_i \leq \Delta_0 + \Delta$
  - Spread-separation probability ($\Delta$) = $(K/N)$
  - Spread-separation-sigma $\sigma_{SS}(\Delta) = \text{std. dev. of } \{(i_{(k+1)} - i_k)\}$
    - Spread-separation-maximum = $\max \{(i_{(k+1)} - i_k)\}$ ; $\text{(max} \approx 4 \cdot \sigma_{SS}(\Delta))$
**TDEV, min_TDEV - definition**

- Given a data set of $N$ points, $\{x_i; i = 0,1,2,...,(N−1)\}$ with implied (equi-spaced) sampling interval $\tau_0$:

  - $TDEV(\tau = n\cdot\tau_0)$ is defined as:

    $$[TDEV(\tau)]^2 = \frac{1}{6(N−3n+1)} \sum_{j=0}^{N−3n} \left[ \frac{1}{n} \sum_{i=j}^{n+j-1} (x_{i+2n} - 2x_{i+n} + x_i) \right]^2$$

  - $\text{min}_TDEV(\tau = n\cdot\tau_0)$ is defined as:

    $$[\text{min}_TDEV(\tau)]^2 = \frac{1}{6(N−3n+1)} \sum_{j=0}^{N−3n} \left[ \min\{(x_{i+2n}); j \leq i \leq (n+j-1)\} - 2 \min\{(x_{i+n}); j \leq i \leq (n+j-1)\} + \min\{(x_i); j \leq i \leq (n+j-1)\} \right]^2$$
Gamma pdf

Synthetic PDV data generated with a gamma pdf for different values of $\alpha$ (1, 2, 3, 4, 5) and $\beta$ chosen to keep standard deviation = 10 $\mu$s. Note that skewness decreases with increasing $\alpha$; a large value of skewness is preferred for timing recovery. Implicit sampling interval is 100ms (10 packets per second).
Spread-separation metrics (gamma pdf)

Spread-separation-sigma provides a clear indication of the skewness of the PDV distribution. Spread-separation-probability provides guidance as to the PDV threshold applicable for packet selection.
min_TDEV for Gamma pdf

The slope of min_TDEV compared to TDEV provides an indication of skewness of the PDV distribution.
Simulation Exercises - pdf

Simulation of 8 switches and varying load (G.8261 Model 1 used for traffic pattern)

10% load

90% load

99% load

50% load

95% load

Clearly, increasing load leads to:

- Increase in mean value
- Increase in variance
- Decrease in skewness

Probability distribution functions for the 5 cases simulated.
Assumed line rate = 1 Gb/s; clock noise (below SEC mask) added
Observation: spread-separation metrics can identify the increase in variance and the decrease in skewness associated with increasing load.
Observation: constant load generates a white-PM packet delay variation. TDEV can identify increase in variance (but not skewness) associated with change in load.
Simulation exercise – varying load

- Load is either 10% or 90% with probability 0.7 and 0.3
- Load changed on 1000s boundary
- TDEV reflects the quasi-periodic behaviour of the PDV

Phase plot showing the change in PDV characteristics on 1000s boundaries
Simulation exercise - min_TDEV

min_TDEV can identify load (variance) as well as indicate skewness.
Impact of frequency offset

Simulation with load = 10% ; added frequency offset = 0ppb, 10ppb, 50ppb, 100ppb.
• TDEV is not affected by frequency offset.
• min_TDEV and spread-separation metrics are affected by frequency offset.
Approximations of min_TDEV

TDEV and min_TDEV computed for scenario with 50% load plus clock noise (below SEC mask)

TDEV shows slope of \(-\frac{1}{2}\)
Indicative of white phase noise

Impact of additive clock noise
Approximations of min_TDEV

- TDEV and min_TDEV are computed on the entire data-set corresponding to a sampling rate of \( f_0 = 1/\tau_0 \).

- Approximations to min_TDEV can be obtained by computing the TDEV on a reduced data-set corresponding to under-sampling by a factor of \( N \):
  - Choose minimum value out of every \( N \) samples.

\[
\{x'(k\tau_0)\} \quad \text{Criterion: minimum} \quad \downarrow N \quad \{x(n\tau_1)\}
\]

- Raw PDV; sampling rate = \( f_0 = 1/\tau_0 \)
- TDEV and min_TDEV computed on a grid of \( \tau_0 \).

- Raw PDV; sampling rate = \( f_1 = 1/\tau_1 = f_0/N \)
- TDEV computed on a grid of \( \tau_1 = N \cdot \tau_0 \).
Approximations of min_TDEV

Under-sampling factor = 5

τ₀ = 100ms;  τ₁ = 500ms
Approximations of min_TDEV

Under-sampling factor = 10
\( \tau_0 = 100\text{ms}; \tau_1 = 1000\text{ms} \)
Approximations of min_TDEV

Under-sampling factor = 30
\[ \tau_0 = 100\text{ms}; \quad \tau_1 = 3000\text{ms} \]
Approximations of min_TDEV

Under-sampling factor = 100

\[ \tau_0 = 100\text{ms}; \tau_1 = 10000\text{ms} \]
Approximations of min_TDEV

Under-sampling factor = 1000
\[ \tau_0 = 100\text{ms}; \tau_1 = 100000\text{ms} \]
Concluding Remarks

- The principal detriment to transferring timing over packet networks is packet delay variation (PDV)
  - Packets with excessive delay variation can be discarded

- Suitability of a packet network for distribution of timing can be quantified using metrics
  - Spread-separation metrics are appropriate when the selected packets are non-uniformly spaced over time
  - TDEV and min_TDEV metrics are suitable when selected packets are chosen over equal duration, contiguous, non-overlapping periods of time

- TDEV and min_TDEV (together) provide useful information on the efficacy of clock recovery methods

- Calculation of min_TDEV can be achieved by computing the TDEV of under-sampled PDV sequences based on a “minimum” criterion